
INTERVAL NOTATION

Consider all the real numbers greater than 3. One simple way to express this set of numbers is the following *inequality*:



$$x > 3$$

[Do you see that this inequality can also be written “ $3 < x$ ”?]]

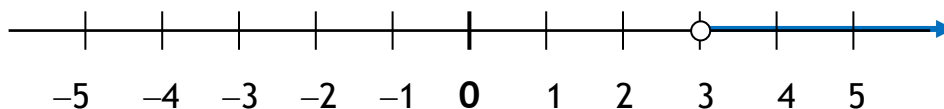
But there are other ways to represent the inequality $x > 3$, ways that might be easier to visualize.

❑ EXAMPLES

First Example: Let's continue with the inequality stated in the introduction:

$$x > 3$$

A great way to visualize this inequality is to **graph** this set of numbers on a number line:



Notice that we put an “open dot” at $x = 3$ to show that the 3 is not part of the set of numbers. But the arrow goes infinitely to the right because $x > 3$ is the set of numbers greater than 3.

Note that the numbers 3.01, π , 17, and 200 are part of the set; but the numbers -5, 0, 2.5, and 3 are not part of the set.

And a third way to denote this interval is called *interval notation*, and for this inequality, we write:

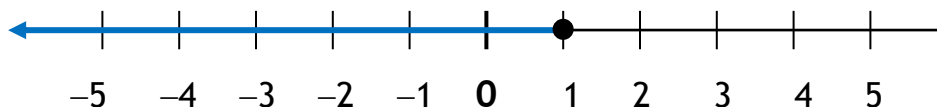
$$(3, \infty)$$

The parenthesis next to the 3 is analogous to the “open dot” on the graph — it means exclude the endpoint. And you always use a parenthesis at the ∞ or $-\infty$ end of an interval because ∞ is not really a number, so you can’t possibly include it.

Second Example: Now we consider all the real numbers less than or equal to 1. This set of numbers can be written as an inequality like this:

$$x \leq 1 \quad \text{Note that this inequality can also be expressed as: } 1 \geq x$$

As a graph on a number line, we write:



The “solid dot” is used to show that $x = 1$ is part of the set of numbers. And since x must be less than or equal to 1, the arrow goes infinitely to the left. Either as an inequality or a graph, you should see that the numbers -3 , -1.1 , $\frac{7}{8}$, and 1 are part of the set, while the numbers 1.001 and $\sqrt{2}$ are not part of the set.

As for interval notation, we write

$$(-\infty, 1]$$

The (square) bracket next to the 1 is analogous to the “solid dot” on the graph — it means include the endpoint. And, as mentioned before, always use a parenthesis with $\pm \infty$, since you can’t ever “get” to ∞ .

Third Example: Now it’s time for an interval that represents all the numbers between two numbers. Consider the double inequality

$$-2 \leq x < 5$$

The inequality can also be read as “all real numbers that are both greater than or equal to 2 AND also less than 5.”

This can be read as “all real numbers between -2 and 5 , including the -2 , but excluding the 5 .”

As for interval notation and a graph, here they are:

$$[-2, 5)$$



Note that some numbers in the interval are -2 , 0 , $\sqrt{24}$, and 4.9999 . But the numbers -2.1 , 5 , and 2π are not in the interval.

Fourth Example: Sometimes answers to an inequality problem end up looking something like this:

$$x < 3 \text{ OR } x \geq 5$$

This means pretty much what it says: x can be less than 3 , or it can be greater than or equal to 5 . As long as x satisfies (at least) one of the two conditions, it's part of the answer. So some x 's that satisfy the inequality are 0 , 2.9 , 5 , and 3π . On the other hand, some numbers that do NOT satisfy it are 3 , π , and 4.99 .

How do we express this in interval notation? Like this, using the **union** symbol:

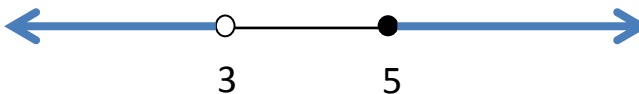
$$(-\infty, 3) \cup [5, \infty)$$

The *union* symbol, $\cup$, just means putting all the elements of the two sets into one big set; for example,

$$\{a, b\} \cup \{a, c, d\} = \{a, b, c, d\}$$

Note that the “overlap” of ‘a’ is listed only once in the final set.

And as a graph, we write:



Can you see that the interval $(-\infty, \infty)$ is the same as the entire set of Real numbers?

$$(-\infty, \infty) = \mathbb{R}$$

In fact, using the *union* notation, we can also write

$$\mathbb{R} = \text{Rationals} \cup \text{Irrationals}$$

where the Rational numbers are all decimals that do repeat, while the Irrational numbers are the decimals that do not repeat. So $\mathbb{R}$ is simply the set of ALL decimals.

□ SUMMARY OF INEQUALITIES AND INTERVALS

Inequality	Interval
$x > b$	(b, ∞)
$x \geq b$	$[b, \infty)$
$x < a$	$(-\infty, a)$
$x \leq a$	$(-\infty, a]$
$a \leq x \leq b$ which means $x \geq a$ AND $x \leq b$, that is, x is <i>between</i> a and b , including both the a and the b .	$[a, b]$
$x < a$ OR $x > b$	$(-\infty, a) \cup (b, \infty)$

Homework

1. Convert each inequality to *interval notation*:
 - a. $x > 2$ b. $x \leq 5$ c. $-1 \leq x < 6$ d. $x < -3$ OR $x > 0$
2. Convert each interval to an *inequality*:
 - a. $[3, \infty)$ b. $(-\infty, -5)$ c. $(-1, 8]$ d. $(-\infty, -2] \cup (7, \infty)$
3. Prove that the statement

$$x < 3 \text{ AND } x \geq 5$$
 consists of absolutely nothing, the null set, $\emptyset$.
4. What is the relationship between the notation $\mathbb{R}$ and the notation $(-\infty, \infty)$?

Solutions

1. a. $(2, \infty)$ b. $(-\infty, 5]$ c. $[-1, 6)$ d. $(-\infty, -3) \cup (0, \infty)$
2. a. $x \geq 3$ b. $x < -5$ c. $-1 < x \leq 8$ d. $x \leq -2$ OR $x > 7$
3. There is no number that is both less than 3 and greater than or equal to 5. The two sets of numbers are disjoint, and so their intersection is $\emptyset$.
4. They both mean exactly the same thing: the set of all **real numbers**.

**“Develop a passion for learning.
If you do, you will never
cease to grow.”**

Anthony J. D'Angelo